The plasma frequency as the boundary between plasma physics and single-particle electrodynamics

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Plasma frequency:

$$\omega_p = \left(\frac{4\pi n_e \; e^2}{m_e}\right)^{\frac{1}{2}}$$

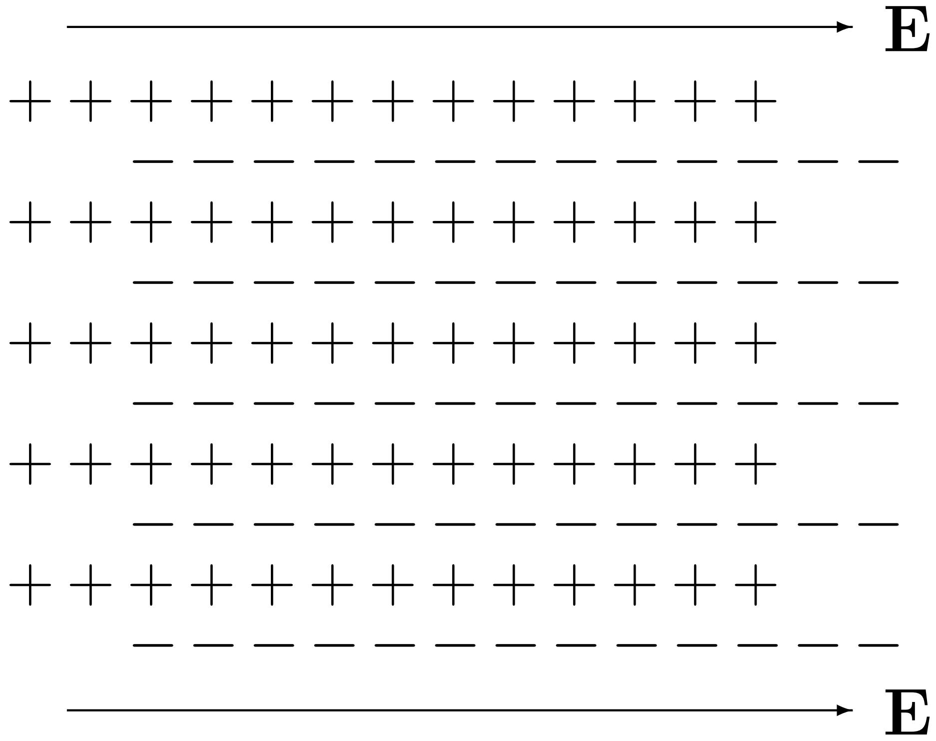
$$= 5.641 \times 10^4 \, \mathrm{s}^{-1} \sqrt{\frac{n_e}{1\,\mathrm{cm}^{-3}}}$$

$$= 2\pi \times 8.978 \text{ kHz} \sqrt{\frac{n_e}{1\,\mathrm{cm}^{-3}}}$$

Electron inertial length (collisionless skin depth):

$$\lambda_e = \frac{c}{\omega_p} = 5.3 \text{ km} \sqrt{\frac{n_e}{1\,\mathrm{cm}^{-3}}}$$

Simple derivation: attempt to separate charges leads to oscillations



$$\frac{\partial}{\partial t} \nabla \cdot \mathbf{E} = -4\pi \ \nabla \cdot \mathbf{J} \qquad\qquad \frac{\partial \mathbf{J}}{\partial t} \simeq \frac{n_e \ e^2}{m_e} \, \mathbf{E}$$

implies $$\frac{\partial^2 \mathbf{E}}{\partial t^2} + {\omega_p}^2\,\mathbf{E} = 0 \qquad \frac{\partial^2 \mathbf{J}}{\partial t^2} + {\omega_p}^2\,\mathbf{J} = 0$$

if assume $$4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} \simeq 0$$ **(electrostatic approximation)**

(plasma frequency intimately connected with the displacement current term in Maxwell's equations).

Look at exact equations:

$$4\pi\mathbf{J} + \frac{\partial \mathbf{E}}{\partial t} = c\nabla\times\mathbf{B} \qquad \frac{\partial \mathbf{B}}{\partial t} = -c\nabla\times\mathbf{E}$$

$$\frac{\partial \mathbf{J}}{\partial t} = \ldots$$ **(sum over all charged particle motions)**

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Definition of current density: $\mathbf{J} = \sum_a q_a \int \ d^3v \ \mathbf{v} f_a(\mathbf{v})$

Generalized Ohm's law:

$$\frac{\partial \mathbf{J}}{\partial t} = \sum_a \left\{ \frac{q_a^2 n_a}{m_a} \left(\mathbf{E} + \frac{\mathbf{V_a}}{c} \times \mathbf{B} \right) - \frac{q_a}{m_a} (\nabla \cdot \boldsymbol{\kappa}_a) + q_a n_a \mathbf{g} \right\} + \left(\frac{\delta \mathbf{J}}{\delta t} \right)_{coll}$$

(averaged over fluctuations)

$$\frac{\partial \langle \mathbf{J} \rangle}{\partial t} = \sum_a \left\{ \frac{q_a^2 \langle n_a \rangle}{m_a} \left(\langle \mathbf{E} \rangle + \frac{\langle \mathbf{V_a} \rangle}{c} \times \langle \mathbf{B} \rangle \right) - \ \frac{q_a}{m_a} \nabla \cdot \langle \boldsymbol{\kappa}_a \rangle + \ q_a \langle n_a \rangle \mathbf{g} \right.$$

$$\left. + \ \frac{q_a^2}{m_a} \left(\langle \delta n_a \delta \mathbf{E} \rangle + \langle \frac{\delta (n_a \mathbf{V_a})}{c} \times \delta \mathbf{B} \rangle \right) \right\} + \ \langle \left(\frac{\delta \mathbf{J}}{\delta t} \right)_{coll} \rangle$$

Plasma momentum equation:

$$\frac{\partial \rho \mathbf{V}}{\partial t} = -\nabla \cdot \boldsymbol{\kappa} + \frac{1}{c} \mathbf{J} \times \mathbf{B} + \left(\frac{\delta \rho \mathbf{V}}{\delta t} \right)_{coll}$$

Key equations (simplified overview):

$$\partial \mathbf{E}/\partial t = -4\pi \mathbf{J} + c\nabla \times \mathbf{B} \tag{1}$$

difference between J & $(c/4\pi)\nabla \times \mathbf{B}$ produces change of E.

$$\partial \mathbf{J}/\partial t = \left({\omega_p}^2/4\pi\right)\left[\mathbf{E} + \mathbf{V} \times \mathbf{B}/c - \mathbf{J} \times \mathbf{B}/\left(n_e e c\right)\right] + \ldots \tag{2}$$

deviation of E from value given by generalized Ohm's law produces change of J, on time scale $\sim {\omega_p}^{-1}$.

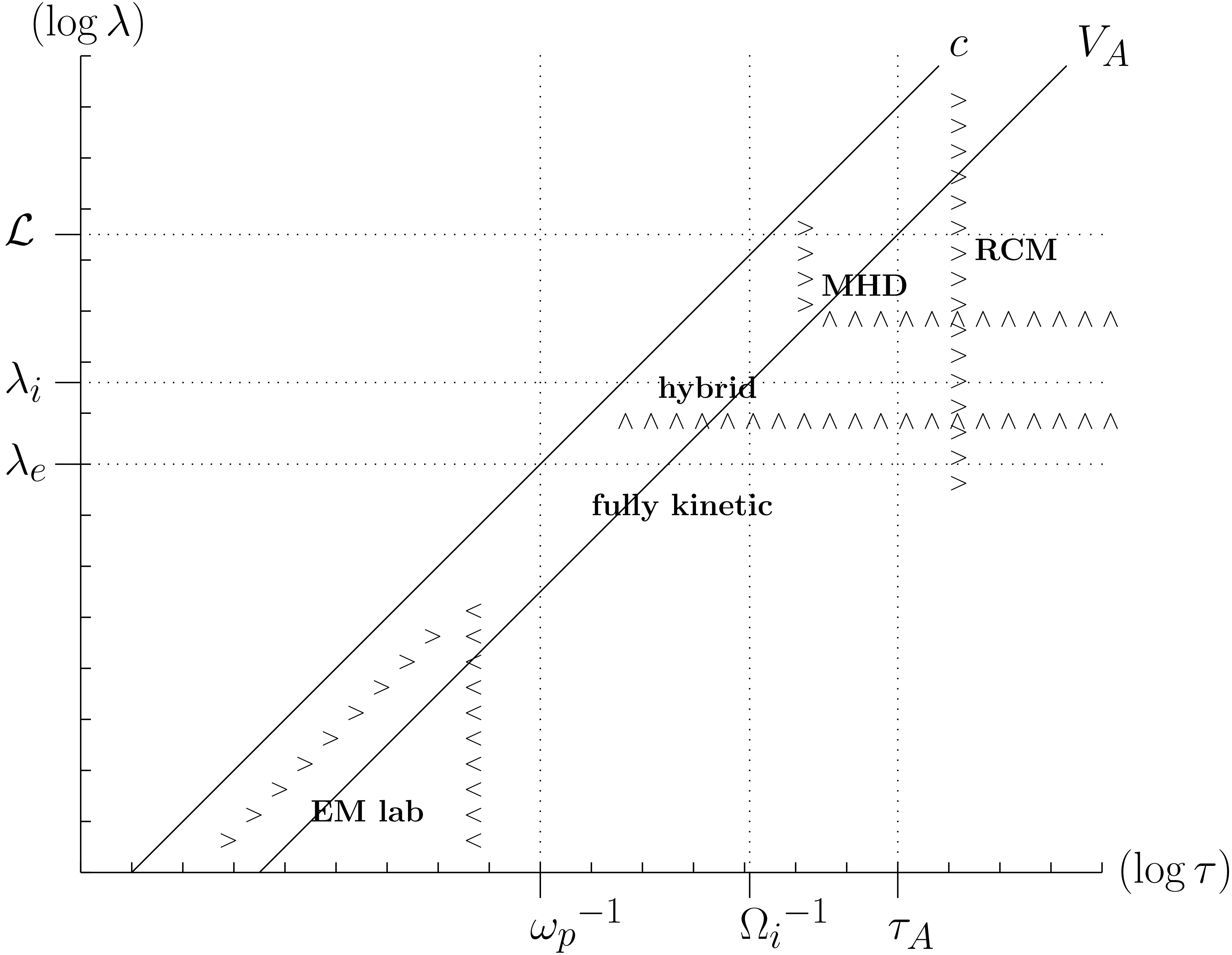
$$\partial \mathbf{B}/\partial t = -c\nabla \times \mathbf{E} \tag{3}$$

change of B produced <u>only</u> if there is spatial variation.

$$\partial \rho \mathbf{V}/\partial t + \ldots = \mathbf{J} \times \mathbf{B}/c - \nu_{pn}\,\rho\left(\mathbf{V} - \mathbf{V}_n\right) \tag{4}$$

change of bulk flow produced only by stress imbalance.

- **J is determined by the motion of <u>all</u> the charged particles, and there is no *a priori* reason why it should equal $(c/4\pi)\nabla \times$ B.**
- **The equality is established through the displacement-current term.**
- **In a large-scale plasma ($\omega_p \tau \gg 1$, $\mathcal{L}/\lambda_e \gg 1$), this occurs primarily by changing J to match the existing $(c/4\pi)\nabla \times$ B, on time scale of order $\sim \omega_p{}^{-1}$.**
- **$\nabla \times$ B changes as consequence of changing B to achieve stress balance, on time scale typically of order $\sim \mathcal{L}/V_A$.**
- **It is only on scales ($\omega_p \tau \ll 1$, $\mathcal{L}/\lambda_e \ll 1$) that primarily $(c/4\pi)\nabla \times$ B changes to match the existing J, on time scale of order $\sim \mathcal{L}/c$, and B may be viewed as determined by a given J.**


$(\log \lambda)$
c
V_A
$\mathcal{L}$
λ_i
λ_e
RCM
MHD
hybrid
fully kinetic
EM lab
$(\log \tau)$
$\omega_p{}^{-1}$
$\Omega_i{}^{-1}$
τ_A

Generalized Ohm's law: $$\frac{\partial \mathbf{J}}{\partial t} = \frac{{\omega_p}^2}{4\pi}\left(\mathbf{E} - \mathbf{E}^*\right)$$

From Maxwell's equations: $$\frac{\partial \mathbf{J}}{\partial t} = -\frac{c^2}{4\pi}\,\nabla \times \nabla \times \mathbf{E} - \frac{1}{4\pi}\,\frac{\partial^2 \mathbf{E}}{\partial t^2}$$

Take curl: $$\frac{\partial}{\partial t}\nabla \times \mathbf{J} = -\frac{{\omega_p}^2}{4\pi}\left(\frac{1}{c}\,\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E}^*\right)$$

$$\frac{\partial}{\partial t}\nabla \times \mathbf{J} = \frac{c^2}{4\pi}\,\left\{\frac{\partial^2}{\partial t^2}\left(\frac{1}{c}\,\frac{\partial \mathbf{B}}{\partial t}\right) - c^2\,\nabla^2\left(\frac{1}{c}\,\frac{\partial \mathbf{B}}{\partial t}\right)\right\}$$

For phenomena on time scales slower than $1/\omega_p$ **and spatial scales longer than** $\lambda_e \equiv c/\omega_p$ **:**

$$0 \simeq \mathbf{E} - \mathbf{E}^* \qquad\qquad \frac{\partial \mathbf{B}}{\partial t} \simeq -c\;\nabla \times \mathbf{E}^*$$