



# Uncertainty Boundaries of the Electron Density Profiles Deduced from Ionograms

Presented by

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# Electron Density Profile

For each layer, the electron density profile is expressed as

$$h = h_m + \sqrt{g} \sum_{i=1}^5 A_i T_i^*(g)$$

with  $g = \ln \frac{f_p / f_m}{f_s / f_m}$

$$h_s = h_m + \sum_{i=1}^5 A_i$$

where

$T_i^*$  = i-th order Chebyshev polynomial

$(h, f_p)$  = (height, plasma frequency)

$(h_m, f_m)$  = (height, plasma frequency)

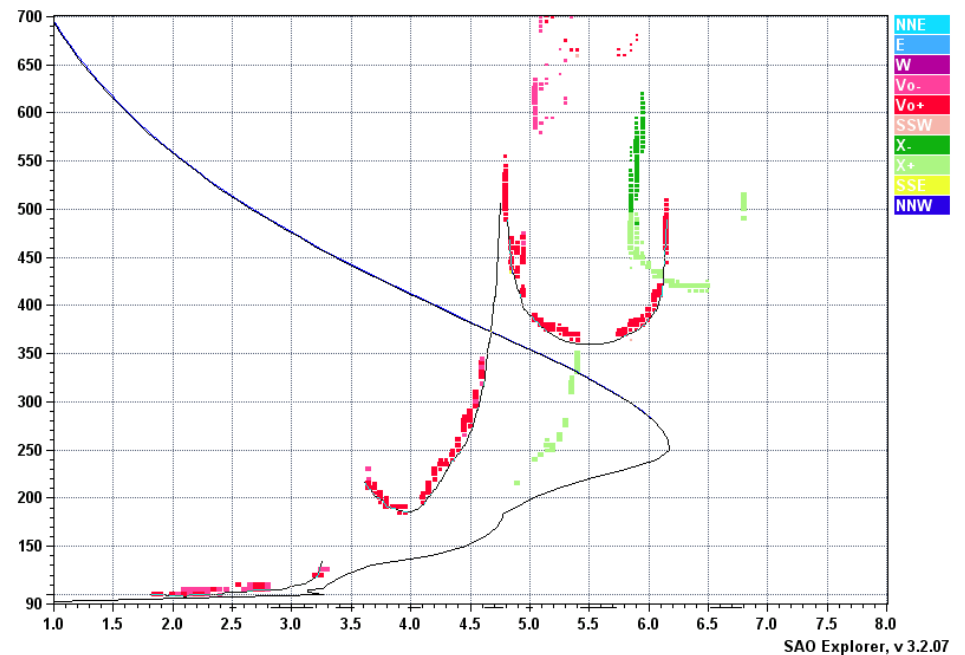
at the layer peak

$(h_s, f_s)$  = (height, plasma frequency)

at the layer bottom

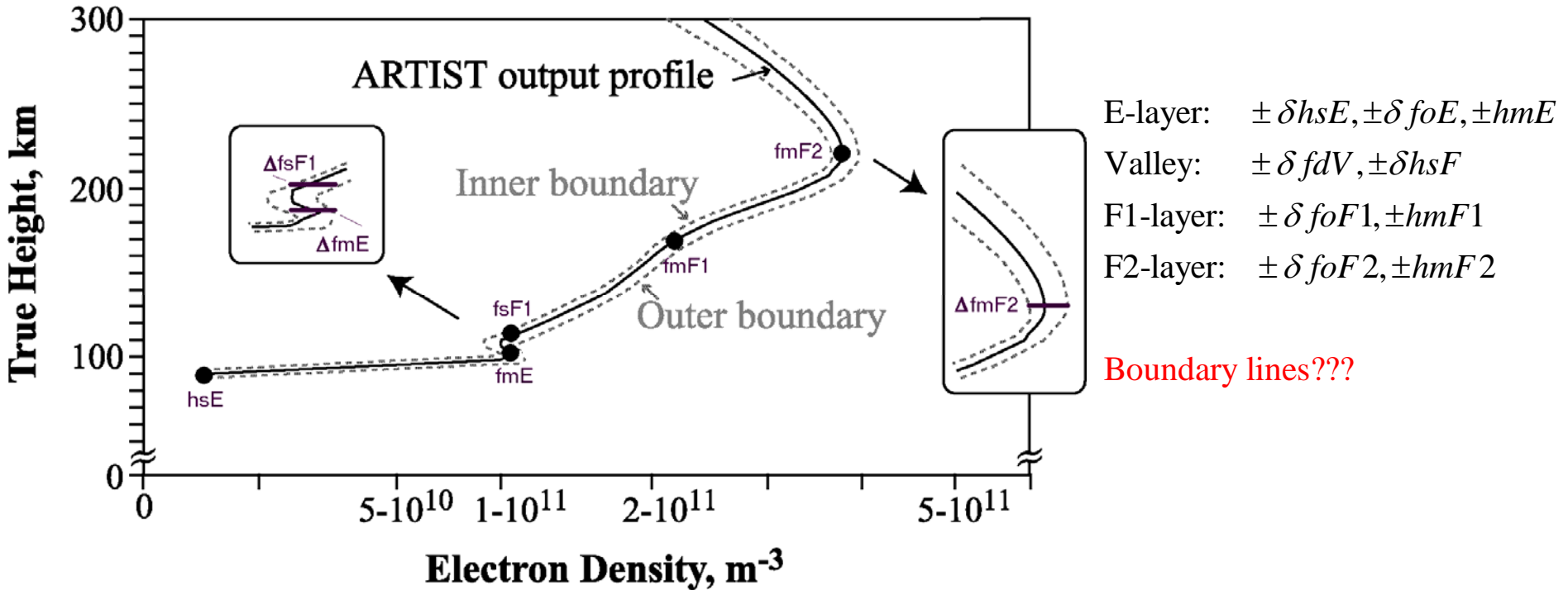
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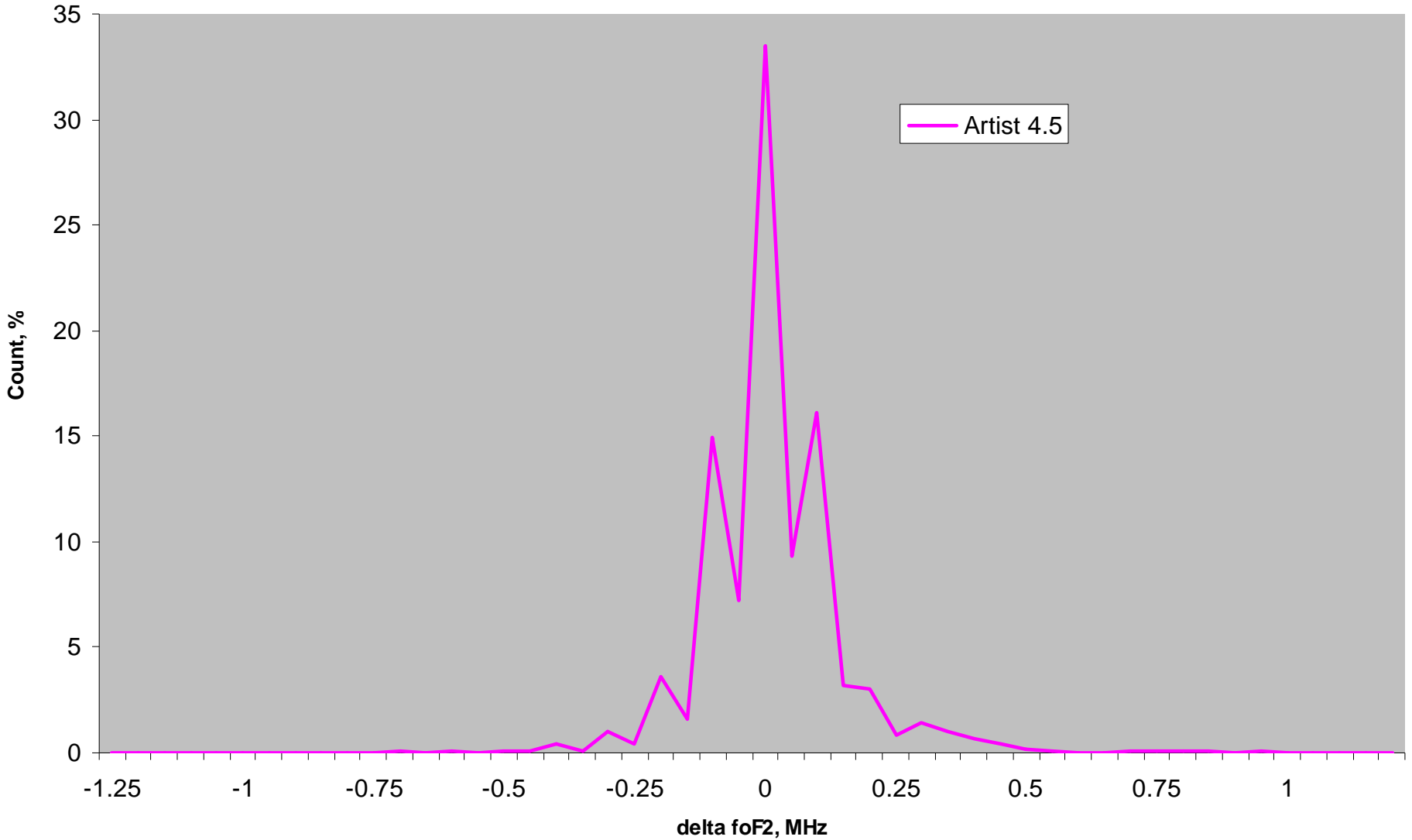
- Deduction method
  - Starting height.
  - Modeled valley.
  - Other assumptions: Horizontal stratification, etc.
- Limited accuracy of the auto-scaling processing.
- Ionogram
  - Spread condition (range/frequency spread).
  - Es blanketing.
  - Ambiguous leading edge, especially caused by low S/N ratio.
  - Improper Digisonde settings.
- Limitations of HF sounding technique

# Uncertainty parameters



	<b>quantity</b>	<b>%</b>	<b>lower</b>	<b>upper</b>	<b>units</b>
<b>foF2</b>	2734	85.9	-0.2	+0.2	MHz
<b>hmF2</b>	2713	78	-20	+20	km
<b>foF1</b>	306	80	-0.35	+0.35	MHz
<b>hmF1</b>	294	83	-20	+20	km
<b>foE</b>	462	88	-0.35	+0.35	MHz
<b>hmE</b>	558	92	-15	+10	km
<b>hsF</b>	2728	80	-5	+5	km
<b>hsE</b>	2732	93	-3	+5	km
<b>foV</b>	2764	87	-0.15	+0.15	MHz

## Boulder, 2004, foF2, manual - auto difference



## Construction of the boundary line

- The boundary lines show the uncertainty range of the deduced density profiles.
- Two kind of boundary lines
  - 50% range: the boundary range in which the profile will fall in with 50% probability.
  - 100% range: the boundary range to show the profile will never be located beyond.
- Construction of boundary lines.
  - Reasonability: No crossing; Smoothly varying.
  - Principle: Parallel to the profile in the g-domain.

## Construction of the boundary line

The parameters of electron density profile for a layer:

$f_s$  : The plasma frequency at the layer starting height.

$f_m$  : The plasma frequency at the layer peak.

$h_m$  : The layer peak height.

$(A_0, A_1, A_2, A_3, A_4)$  : The coefficients of the shifted Chebyshev polynomial.

$$h = h_m + \sqrt{g} \sum_i A_i T_i^*(g), \quad g = \frac{\ln(f_p / f_m)}{\ln(f_s / f_m)}, \quad f_s \leq f_p \leq f_m$$

The parameters of a boundary line:

$f'_s$  : The plasma frequency at the starting height.

$f'_m$  : The plasma frequency at the boundary peak.

$h'_m$  : The boundary peak height.

$(B_0, B_1, B_2, B_3, B_4)$  : The coefficients of the shifted Chebyshev polynomial.

$$h = h'_m + \sqrt{g} \sum_i A_i T_i^*(g), \quad g = \frac{\ln(f_p / f'_m)}{\ln(f'_s / f'_m)}, \quad f'_s \leq f_p \leq f'_m$$

# Construction of the boundary line

- The boundary lines:

$$\begin{bmatrix} B_0 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \sqrt{\lambda} \mathbf{R}_2 \mathbf{R}_1 \begin{bmatrix} A_0 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{bmatrix},$$

$$\lambda = \frac{\ln\left(\frac{f'_s}{f'_m}\right)}{\ln\left(\frac{f_s}{f_m}\right)},$$

$$\mathbf{R}_1 = \begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 0 & 2 & -8 & 18 & -32 \\ 0 & 0 & 8 & -48 & 160 \\ 0 & 0 & 0 & 32 & -256 \\ 0 & 0 & 0 & 0 & 128 \end{bmatrix},$$

$$\mathbf{R}_2 = \begin{bmatrix} 1 & \lambda/2 & 3\lambda^2/8 & 10\lambda^3/32 & 35\lambda^4/128 \\ 0 & \lambda/2 & 4\lambda^2/8 & 15\lambda^3/32 & 56\lambda^4/128 \\ 0 & 0 & \lambda^2/8 & 6\lambda^3/32 & 28\lambda^4/128 \\ 0 & 0 & 0 & \lambda^3/32 & 8\lambda^4/128 \\ 0 & 0 & 0 & 0 & \lambda^4/128 \end{bmatrix}$$

- Regulate the truncated  $B$ 's to allow the continuity at the starting height:

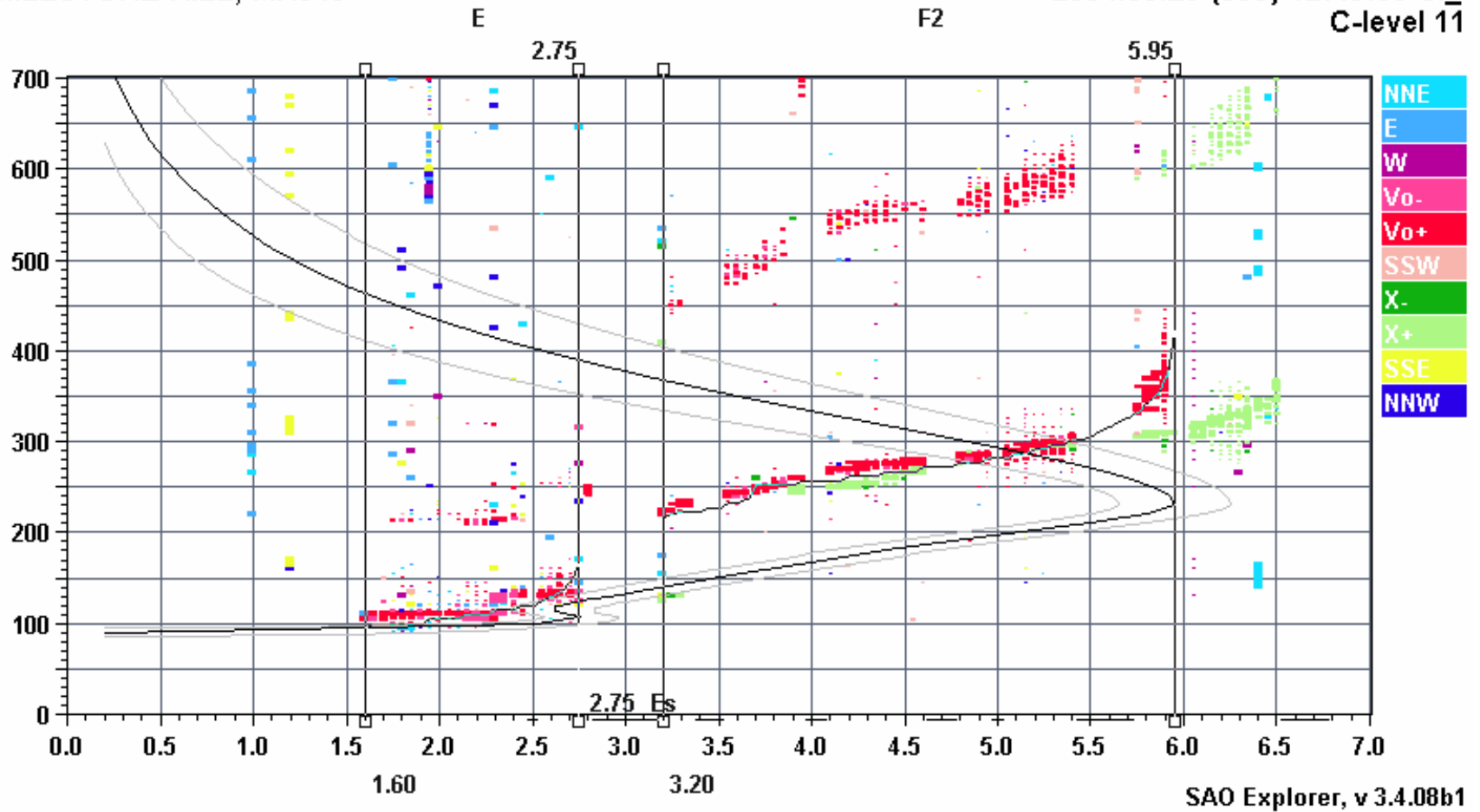
$$h'_m \Big|_{\text{lower layer}} = h'_s \Big|_{\text{current layer}} = h'_m \Big|_{\text{current layer}} + \sum_i B_i$$

- For the valley, the treatment is similar.

# Density profile uncertainty boundary

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## Conclusions

- Uncertainty of derived density profiles is estimated.
- This uncertainty data is important for the study of the ionospheric density structure: Modeling, Mapping, etc.